EXPLOITING THE CHARACTERISTICS OF CENTRAL COMPOSITE DESIGN FOR HIGH YIELD OF SORGHUM

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ABSTRACT
Sorghum is a nutritious cereal grain with a potential for healthy feeding, regional development and sustainable economies. The World Food Programme (WFP), Sustainable Development Goal 2 (SDG): Zero hunger, aims at eradicating hunger in the world. Maximizing yield of Sorghum by exploiting the statistical Central Composite Design (CCD) characteristics would be attractive in meeting the need to eradicate hanger. This paper focuses on using one Response Surface Methodology (RSM), the CCD to develop a model that is useful in high production of Sorghum in Meru County, Kenya using application of organic and inorganic fertilizers. This study used n*itrogen fertilizer (* X_1 *)* supplied in form of Calcium Ammonium Nitrate (CAN) 26%, goat manure (X_2) and vegimax folia fertilizer (X_3 *)*.as the independent variables The data obtained from the study was analyzed using the R statistical software and the output showed that Nitrogen fertilizer, Goat manure and Vegimax folia were significant at varied levels of significance ($p < 0.001$, P=0.0485249, and P=0.0096342 respectively). All the quadratic functions were significant (p-values < 0.05) but the factor interactions were all insignificant (p-value>0.05). The findings indicated that 112 Kg/ha of Nitrogen fertilizer, 12 tons/ha of goat manure and 5.5 ml/20L(of water) of Vegimax folia were the optimum values that lead to maximum sorghum grain yield of 4.95 tons/ha. The Multiple R-squared and Adjusted R-squared were 0.9277 and 0.8627 respectively, an indication that the fitted model explained 92.8% and 86.3% of the variations in the model. The research output will contribute to sustainable livelihoods and economies for transformative societies.

Key words: Sorghum, Response Surface Methodology, Optimization, Organic and Inorganic fertilizer

INTRODUCTION
The Response Surface Methodology (RSM) is a collection of statistical experiment designs that are used for modeling and issue analysis when a response of interest is affected by a wide range of variables. The goal of RSM is to find the independent variables' ideal settings in order to maximize the response (Myers, 2004). In order to properly conduct experiments through adequate design and to establish operating conditions on a set of controllable variables that yield an ideal response, Box and Wilson (1951) invented the Response Surface Methodology. The RSM enables the creation of an empirical model that makes a response surface a function of the input variables and enables the determination of a more precise approximation of the process' ideal operating conditions.

RSM allows the development of an empirical model that makes a response surface a function of the input variables and enables the determination of a more precise approximation of the process' ideal operating conditions. In general, in a situation where is the response variable of interest and there is a set of predictor variables,, that influence the response variable, the relationship between the response and the set of independent variables is given as

$$y = f(x_i) + \epsilon \quad ;i = 1,2,\dots,s \qquad (1)$$

The term denoted as ϵ is the error term and represents any measurement error observed in the response y, as well as other types of variations not accounted in the function. It is a statistical error assumed to be normally independent and identically distributed with zero mean and variance (σ^2) of one (Montgomery *et al.,* 2014). The expected response of y in (1) is denoted by $E(y)$ as in equation (2) and represents a surface referred as the response surface.

$$E(y) = f(x_1, x_2, \dots, x_s) = \eta \qquad (2)$$

Two important cases of model (2) commonly used in RSM includes the low-order model $(d = 1)$,

$$y = \beta_0 + \sum_{i=1}^{s} \beta_i x_i + \varepsilon, \qquad i = 1,2,\dots,s \qquad (3)$$

and the second-order model $(d = 2)$

$$y = \beta_0 + \sum_{i}^{s} \beta_i x_i + \sum_{i=1}^{s} \beta_{ii} x_i^2 + \sum\sum_{i<j}^{s-1} \beta_{ij} x_i x_j + \varepsilon\ ;\ ((\ i = 1,2,\dots,s; j = 2,3,\dots,s \text{ and } i < j) \qquad (4)$$

Where β_0 is a constant, β_i is the linear coefficient, β_{ii} is the coefficient of pure quadratic terms and β_{ij} is the coefficient of cross-product terms (interaction)

The initial step in the application of RSM is to establish the true functional relationship between the response variable and the set of predicting variables. The lower- order model (3) serves as an approximation function when the response variable can accurately be described by a linear function of the regressors. The model is purposely used for screening and steepest ascend or descend. In case of the presence of curvature in the response, a higher degree polynomial such as the RSM model (4) is employed in order to obtain the curvature. (Myers, 2016; Khuri, 2010).

RSM designs have been widely applied in a variety of fields, including industrial, biological, clinical, social, food, engineering, and agricultural sciences (Chelule, 2014), to establish a relationship between the input factors and the corresponding responses

Agricultural research studies have revealed the effect of different input factors like fertilizers, rate of irrigation, seed density etc. on sorghum yield (Muui *et al*., 2013; Kagwiria, 2019; Mwadalu *et al*., 2022). However, the studies neither determined the optimal settings for maximum sorghum yield nor developed an empirical model making the yield of the sorghum a function of the input factors. Factors like organic and inorganic fertilizers when added into soil creates ideal growing environment for the crops through improvement of its fertility as sited by (Titirmare *et al.,* 2023) and consequently promote the crop performance. For instance, a study by Kagwiria, (2019) showed that application of fertilizers affects positively sorghum production. However, this study did not attempt to optimize levels of fertilizers applied for maximum production of sorghum grains.

Local industrial and domestic demand for sorghum has increased as a result of the current increase in sorghum consumption as food and a raw material for industry (Wambugu, 2011). This is undoubtedly promoting higher sorghum grain output and improving the livelihood and standard of living of farmers. This means that sorghum production has the potential to promote regional development, as well as the eradication of poverty, the creation of jobs, the fight against hunger, and undoubtedly an effort to realize Vision 2030 for the agricultural sector as well as the Sustainable Development Goals on achieving zero hunger, good health, and wellbeing.

The current study applied Central Composite Design (CCD) as the RSM to develop a model that is useful in high production of Sorghum grain yield in Meru County, Kenya using application of organic and inorganic fertilizers.The general objective of study was to exploite the characteristics of central composite design for high yield of sorghum, and was guided by the following specific objectives;

i. To determine the effect of inorganic and organic fertilizer on the yield sorghum using CCD

To determine a functional higher order polynomial model that best fits experimental data

To determine the optimal values of organic and inorganic fertilizers that leads maximum sorghum yield.

MATERIAL AND METHOD

Material

SC-Sila- a hybrid sorghum variety developed by East Africa Seed Company was used in the study. Nitrogen fertilizer (80 Kg/ha, 100 Kg/ha, 120 Kg/ha) supplied in form of Calcium Ammonium Nitrate 26%, Goat manure (5 tons/ha, 10 tons/ha,15 tons/ha) and VEGIMAX Folia fertilizer (4 ml/20 L, 5 ml/20 L, 6 ml/20 L) were the primary input factors used for the experiment. Escort 50 EC and Integra were the only pesticides used.

Experiment

The experiment used twenty plots of 2 M by 1.25 M each with no blocking and each block randomly assigned to each run for the respective treatment. Each plot had two rows with ten seed holes per row and seed density of two per seed hole. The experiment used an inter row spacing of 75 cm and seed hole spacing of 20 cm along the row. Before the seed was covered, the soil and goat dung were thoroughly mixed. First weeding was done at day ten after emergence and at day fifteen, thinning was done to ensure a uniform seed density of two per hole. Application of vegimax folia and first half dose of nitrogen fertilizer were administered the same day. After the second weeding, a second administration of vegimax folia and the second half dose of nitrogen fertilizer was done at the thirtieth day after emergence.

Pesticide such as Escort 50 EC at the rate of 20-25 ml plus Integra 3 ml in 20 litres of water were used to manage pests such as fall armyworms. SC-sila- a hybrid sorghum variety from a certified seed company was also used in the experiment. Since the experiment was carried out in only one ecological zone, the study assumed homogeneity of soils in the twenty plots with other factors such as temperatures and humidity considered constant.

Data on the weight of sorghum grain yield per run was collected after harvesting and sun drying the harvested sorghum heads at maturity from the experiment carried out during long rains at a farm in Gakumbo Village within Mbirikene Sub-Location in Meru County.

Design of Experiment

A 5-level-3-factor Central Composite Design was employed as the RSM in the study experiment comprising of 20 experimental runs determined by the 2^3 full factorial design with six axial points and six center points. The variance of the fitted response at different points of interest was kept constant and stable via a rotatable and orthogonal CCD Nitrogen fertilizer (X_1) supplied in form of CAN 26%, goat manure (X_2) and VEGIMAX foliar (X_3) were the input variables used in the experiment to optimize the sorghum grain yield. The design was centered about the current operating conditions of the system given as 100kg ha^{-1}, 10 tons ha^{-1} and 5ml/20L of water for Nitrogen fertilizer supplied in form of CAN, goat manure and VEGIMAX foliar respectively. Therefore the region of exploration for this study was (80-120) kg ha^{-1} of nitrogen fertilizer, (5-15) tonnes ha^{-1} of goat manure and (4-6) ml / 20 litres of water of vegimax folia.

Three Factor Central Composite Design

The CCD was centered about the current operating conditions of the system and for development of the mathematical model the input variables were coded to (-1,1) interval. The respective coded variables were obtained using equation (5)

$$x_i = \frac{X_i - X_0}{\omega}$$

Letting X_i is the i^{th} actual variable, x_i is the coded variable, X_0 is the average of the high and low level of the i^{th} variable and ω is half of the difference between the levels of the i^{th} variable Then in reference to equation (5) the coded variable x_i for *i*= 1,2,3 were

$$x_1 = \frac{X_1 - 100}{20}$$

$$x_2 = \frac{X_2 - 10}{5}$$

and

$$x_3 = \frac{X_3 - 5}{1}$$

The i^{th} actual variables X_i for $i = 1,2,3$ corresponding to the axial variable coded $\pm\alpha$ were obtained using the equation (6)

$$X_i|_{\pm\alpha} = \tau_i \pm \alpha\omega \quad (6)$$

Where τ_i is the current operating condition for the i^{th} variable and ω is half of the difference between the levels of the i^{th} variable.In reference to equation (6), the actual variables X_i corresponding to the axial variable coded $\pm\alpha$ for this study were

$X_1|_{\pm\alpha} = 100 \pm 1.682(20)$, $X_2|_{\pm\alpha} = 10 \pm 1.682(5)$ and $X_3|_{\pm\alpha} = 5 \pm 1.682(1)$

Table 1 presents the three experimental actual variables used in the study together with their coded factors at five levels.

Table 1: The three actual variables and their coded factors at five levels

Independent variable	Coded factor levels				
	-1.682	-1	0	1	1.682
Nitrogen fertilizer (x_1) Kg ha^{-1}	66.36	80	100	120	133.64
Goat manure (x_2) tons ha^{-1}	1.59	5	10	15	18.41
VEGIMAX folia (x_3) ml/20L	3.318	4	5	6	6.682

Source: Author conceptualization

Data collection

Data on the weight of sorghum yield per run was collected after harvesting and sun drying the harvested sorghum heads at maturity. This was done by measuring the weight of total sorghum grain yield (TSGY) and the weight of thousand grains (WTG) per run separately in grams using a laboratory electronic balance and recorded to one decimal point. Table 2 presents the CCD modal data on sorghum yield per run collected from the experiment. The weight of the thousand grains of sorghum in grams and the weight of the total yield of sorghum grains per run are the two coded and actual values for the three independent variables that make up the CCD design matrix.

Data analysis

Model fitting

The study used the data collected on Total Sorghum Grain Yield (TSGY) only to develop a model useful for high sorghum grain yield production in Meru County. The data collected was subjected to analysis using the R software and the use of least squares method was employed in the regression analysis to determine the polynomial model for sorghum grain yield. A second order regression model representing sorghum grain yield as a function of Nitrogen fertilizer (X_1), Goat manure (X_2) and Vegimax folia (X_3) at maturity was expressed as:

$$y = \hat{\beta}_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{33} x_3^2 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 + \varepsilon$$
(7)

Where y is the predicted response, β_0 is a constant, β_i is the linear coefficient, β_{ii} is the coefficient of pure quadratic terms and β_{ij} is the coefficient of cross-product terms and ε is the error term with a mean of zero and a variance of one $(i = 1,2,3; j = 2,3 \ and \ i < j)$. For regression analysis the study expressed the second order model in matrix form as

$$y = \hat{\boldsymbol{\beta}}_0 + \boldsymbol{X}^T \boldsymbol{b} + \boldsymbol{X}^T \boldsymbol{\beta} \boldsymbol{X} + \epsilon$$
(8)

Where $\boldsymbol{b}$ is a (3×1) vector of the first-order regression coefficients and $\boldsymbol{\beta}$ is a (3×3) symmetric matrix whose main diagonal elements are pure quadratic coefficients ($\hat{\beta}_{ii}$) and whose off-diagonal elements are one-half the mixed quadratic coefficients ($\hat{\beta}_{ij}$)

Model adequacy check and validation

To check the adequacy of the model for the response the study used the Analysis of variance (ANOVA) for the regression significance at 95% confidence level. The ANOVA employs the sum of squares and the F-statistic test to identify which process parameter is important (Ranganath, 2015). If the *p*-value for the statistic F_0 is less than $\alpha = 5\%$ level of significance, it implies that at least more than one coefficient differs, that is, model terms are significant. The study used the coefficient of determination of squared R (R^2) and the adjusted squared R ($R^2{}_{Adj}$) to validate the regression model. If R^2 and $R^2{}_{Adj}$ differ greatly, there is a high likelihood of non-significant terms being present in the model. Large values of R^2 and $R^2{}_{Adj}$ that are close to one shows that the model adequately explain the variations in the response surface and therefore the model can approximate the experimental data at the given design points.

Optimization

The optimum levels of Organic and Inorganic fertilizers that optimizes the response (sorghum grain yield) is a set of x_1, x_2, x_3 for which the partial derivatives of the response with respect to x_1, x_2, x_3 are zero.

$$\frac{\partial \hat{y}}{\partial x_1} = \frac{\partial \hat{y}}{\partial x_2} = \frac{\partial \hat{y}}{\partial x_3} = 0 \qquad (9)$$

For second-order model $\hat{y} = \hat{\beta}_0 + X^T b + X^T \beta X$, the derivative of $\hat{y}$ with respect to the elements of vector X is

$$\frac{\partial \hat{y}}{\partial x} = b + 2\beta X = 0 \qquad (10)$$

This condition occurs at the stationary point at which the mathematical solution for the set of x_1, x_2, x_3 is the optimum levels for the input variables that optimized the response. Equation (11) gives the mathematical solution for the set of x_1, x_2, x_3 at the stationary point that yields the optimal values.

$$x_t = \frac{1}{2} \beta^{-1} b \qquad (11)$$

The optimum sorghum grain yield at the stationary point was therefore predicted using the equation (12)

$$\hat{y} = \hat{\beta}_0 + \frac{1}{2} x_t^{-1} \qquad (12)$$

RESULTS AND DISCUSSION

Full Factorial Central Composite Design Matrix and Experimental Results for Sorghum Yield

Data on the weight of sorghum grain yield per run was collected after harvesting and sun drying the harvested sorghum heads at maturity. The weight of total sorghum grain yield (TSGY) and that of thousand grains (WTG) per run were weighed separately in grams using a laboratory electronic balance and recorded to one decimal point. Table 2 presents the Full Factorial CCD Design model data on sorghum yield per run.

Table 2: CCD Model Data on Sorghum Yield per run

	Coded values			Actual values			Main crop Yields (g/run)	
Runs	x1	x2	x3	N (Kg/ha)	GM (tons/ha)	VF (ml/20 L)	WTG (Y1)	TSGY (Y2)
1	-1	-1	-1	80	5	4	19.4	905.6
2	1	-1	-1	120	5	4	21.3	1088.2
3	-1	1	-1	80	15	4	22.4	931.2
4	1	1	-1	120	15	4	25.3	1103.7
5	-1	-1	1	80	5	6	20.6	970.3
6	1	-1	1	120	5	6	22.7	1115.5
7	-1	1	1	80	15	6	25.3	1005.2
8	1	1	1	120	15	6	26.2	1255.1

9	-1.682	0	0	66.36	10	5	18.2	897.4
10	1.682	0	0	133.64	10	5	21.7	1064.7
11	0	-1.682	0	100	1.59	5	20.8	992.5
12	0	1.682	0	100	18.41	5	22.1	1072.7
13	0	0	-1.682	100	10	3.318	22.4	1001.3
14	0	0	1.682	100	10	6.682	25.6	1108.7
15	0	0	0	100	10	5	24.8	1198.2
16	0	0	0	100	10	5	25.2	1223.4
17	0	0	0	100	10	5	24.9	1250.3
18	0	0	0	100	10	5	25.3	1206.6
19	0	0	0	100	10	5	24.4	1150.3
20	0	0	0	100	10	5	25.1	1189.6

Where, N is Nitrogen fertilizer supplied as CAN 26%, GM is the goat manure, VF is the Vegimax Folia, WTG (Y1) is the weight of 1000 grains of sorghum and TSGY (Y2) is the weight of total sorghum grain yield per run

Summary Statistics for the Weight of Sorghum Grain Yield and 1000 Grains of Sorghum in grams per run

Descriptive and normality test statistics for the weight of thousand grains (WTG) grain and total sorghum grain yield (TSGY) per run were carried out and results presented in the Table 3. The result of the study showed that the average weight of 1000 grains of sorghum produced was 23.18 grams with a median of 23.55 grams per run and a standard deviation of 2.33. At the same time, the average weight of sorghum grain yield produced was 1086.5 grams with a median of 1095.95 grams per run and a standard deviation of 113.95.

Table 3: Summary statistics for the weight of sorghum grain yield and 1000 grains of sorghum in grams per run

Response	Mean	Standard deviation	Median	Min	Max	Skewness	Kurtosis
WTG (Y1)	23.18	2.33	23.55	18.20	26.20	-0.50	-1.06
TSGY (Y2)	1086.53	113.95	1095.95	897.40	1255.10	-0.13	-1.32

Where WTG is the weight of 1000 grains of sorghum in grams and TSGY is the weight of sorghum grain yield in grams per run.

According to the study's findings, each run generated 23.18 grams of WTG sorghum grains on average, with a median of 23.55 grams and a standard deviation of 2.33. The weight of TSGY produced overall, with a median of 1095.95 grams per run and a standard deviation of 113.95, was 1086.5 grams on average. For WTG and TSGY, the skewness values were -0.50 and -0.13, respectively. While WTG and TSGY had kurtosis scores of 1.06 and 1.32, respectively. Since the weight of TSGY and WTG of sorghum per run suggested normal distribution of the data since their skewness and kurtosis values were falling within the range of ±3. The findings on this study agrees with the findings of Aczel and Sounderpadian (2002) who attributed that for the normality of data, the skewness should fall within a range of ±3.

Effect of Nitrogen Fertilizer, Goat Manure and Vegimax Folia on Sorghum Yields

The collected data in Table 2 in subsection 3.1 indicated that the treatment given to the 8^{th} run had an average yield of 26.2 grams and 1255.1 grams being the WTG and TSGY respectively. Similarly, the results indicated that treatment given to the 9^{th} run had an average yield of 18.2 grams and 897.4 grams being the WTG and TSGY respectively. The results showed that the treatment combination with 66.36 Kg/ ha of nitrogen, 10 tonnes / ha of goat manure and 5 ml per 20 litres of vegimax had the lowest sorghum grain yield. At the same time, the combination with 120 Kg / ha of nitrogen fertilizer, 15 tonnes / ha of goat manure and the 6 ml per 20 litres of vegimax folia produced the highest sorghum yield. The results further reveals that varying the quantity of nitrogen fertilizer, goat manure and vegimax folia applied on sorghum from low to moderately high level led to increase in sorghum yield. The treatment combination with 120 Kg / ha of nitrogen fertilizer, 15 tonnes / ha of goat manure and the 6 ml per 20 litres of vegimax folia was the most effective treatment among the range of treatments applied. These results reveals that integration of nitrogen fertilizer, goat manure and vegimax folia influence sorghum yield

Model fitting

The study used only the data on TSGY to develop a regression model useful for determining a functional relationship between nitrogen fertilizer, goat manure and sorghum grain yields within the area of study. The study used the R software to compute the parameter estimates for a Second order RSM model for TSGY per run and the output were presented in table 4.

Table 4: Second Order RSM Model for TSGY (weight in gram)

Factors	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	1201.946	17.223	69.7857	8.899e-15 ***
x1	75.529	11.427	6.6099	6.003e-05 ***
x2	25.662	11.427	2.2458	0.0485249 *
x3	36.465	11.427	3.1912	0.0096342 **
x1:x2	11.825	14.930	0.7920	0.4467363
x1:x3	5.000	14.930	0.3349	0.7446286
x2:x3	16.675	14.930	1.1168	0.2901743
x1^2	-71.124	11.122	-6.3949	7.884e-05 ***
x2^2	-52.903	11.122	-4.7566	0.0007725 ***
x3^2	-44.985	11.122	-4.0447	0.0023434 **

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1; Multiple R-squared: 0.9277, Adjusted R-squared: 0.8627; F-statistic: 14.26 on 9 and 10 DF, p-value: 0.0001358

The regression model output (table 4) represents a second order model for high production of sorghum grain yield and was presented using a mathematical equation (13)

$$y = 1201.946 + 75.529x_1 + 25.662x_2 + 36.465x_3 + 11.825x_1x_2 + 5.000x_1x_3 + 16.675x_2x_3 - 71.124x_1^2 - 52.903x_2^2 - 44.985x_3^2$$

This is the model useful for determining a functional relationship between nitrogen fertilizer, goat manure, vegimax folia and sorghum grain yields within Gakumbombo village in Meru County. Where y is the total sorghum grain yield in grams; $x_1 =$ Nitrogen fertilizer supplied inform of CAN 26%; $x_2 =$ Goat manure and $x_3 =$ Vegimax folia.

The model output showed that the three predictors were significant at varied levels of significance with the p-values for Nitrogen fertilizer, Goat manure and Vegimax folia being < 0.0001, $<$ 0.05, and $<$ 0.01 respectively. The model output shows that any unit increase in Nitrogen fertilizer, Goat manure and Vegimax folia leads to a corresponding increase of 75.529, 25.662 and 36.465 in the weight of TSGY.. As a result, higher quantities of nitrogen fertilizer, goat manure, and vegimax folia would boost sorghum grain output. The model's F-statistic of 14.26 on degrees of freedom 9 and 10 was higher at 5% level of significance than the critical F value of 3.02 on the same degrees of freedom. This indicated that there was a correlation between the sorghum grain yield and the three independent factors. An assertion that a major portion of the fluctuation in the sorghum grain production was explained by nitrogen fertilizer, goat manure and vegimax folia.

The findings on this study agrees with the findings of Kimtai (2019) who observed a positive response of organic manure (p < 0.05) to common bean performance. From the study Muriithi (2015), the findings revealed that application of Nitrogen and Phosphorus fertilizers led to an increased productivity of potatoes tubers. The study by Mwadalu *et al*.,(2022) also revealed that the application of organic and inorganic fertilizers has a significant effect on the sorghum response. The interaction effects of the three predictors on the sorghum grain yield were found to be insignificant with their p- values all $>$ 0.05. The model output also indicated that the quadratic functions for Nitrogen fertilizer (p-value < 0.001), Goat manure (p-value < 0.001) and Vegimax folia (p-value $<$ 0.01) were statistically significant with their p-values all less than 0.05. Any unit increase in the quadratic functions for Nitrogen fertilizer, Goat manure and Vegimax folia leads to a corresponding decrease of 71.124, 52.903 and 44.985 respectively in the weight of sorghum grain yield.

Model adequacy check and validation

To check the adequacy of the model for the response the study used the Analysis of variance (ANOVA) for the regression significance at 95% confidence level and the ANOVA output were presented in ANOVA table 5

Table 5: ANOVA for the Second Order model for TSGY per run

Source of variance	df	SS	MS	F value	p-value
FO(x1, x2, x3)	3	105072	35024	19.6396	0.0001632
TWI(x1, x2, x3)	3	3543	1181	0.6623	0.5938769
PQ(x1, x2, x3)	3	120258	40086	22.4781	9.185e-05
Residuals	10	17833			
Lack of fit	5	12187	2437	2.1584	0.2091840
Pure error	5	5646	1129		

The ANOVA results provided in Table 5 includes; the first order model (FO), a study of the response surface with two-way interactions (TWI), and pure quadratic terms (PQ). The outcome also demonstrates the second-order model's lack of fit and obvious error.. The ANOVA output showed that the lack of fit was insignificant (p-value =0.2091840 > 0.05) with F(5,5) = 2.1584 whereas the important F value for the same degrees of freedom was 5.05, making the model significant. Therefore, the study concludes that the model is adequate and becomes significant model for predicting sorghum grain output when nitrogen fertilizer, goat manure and vegimax folia are applied. The output (table 4) indicated that at a confidence level of 0.95, the R^2 and $R^2{}_{Adj}$ were 0.9277 and 0.8627 respectively, an indication that the fitted model explained 86.3% of the variations in the model.

Optimization

Stationary Points

The stationary point of the response surface was computed using the R-statistical software and the output presented in Table 6.

Table 6: Stationary point of the response surface

x_1	x_2	x_3
0.5811102	0.3877729	0.5094602

The response surface stationary points were in coded form and were transformed to actual values through the equations:

$x_1 = \frac{X_1 - 100}{20}$, $x_2 = \frac{X_2 - 10}{5}$ and $x_3 = \frac{X_3 - 5}{1}$

Such that $X_1 = 112$ Kg / ha, $X_2 = 12$ tonnes / ha and $X_3 = 5.5$ ml / 20 litres of water.

The stationary points could also be calculated manually in reference to equation (11) as discussed below. The $(s \times 1)$ vector b and the $(s \times s)$ symmetric matrix β^{-1} needed for the analysis of the stationary point were given as

$$\beta = \begin{bmatrix} -71.124 & 5.913 & 2.5 \\ 5.913 & -52.903 & 8.338 \\ 2.5 & 8.338 & -44.985 \end{bmatrix}$$

$$\det \beta = -162169.14$$

$$C^T = \begin{bmatrix} 2310.32 & 286.84 & 181.56 \\ 286.84 & 3193.26 & 607.81 \\ 181.56 & 607.81 & 3727.71 \end{bmatrix}$$

where C^T is the transpose for matrix of cofactors.

$$\beta^{-1} = \frac{1}{\det \beta} C^T = \frac{1}{-162169.14} \begin{bmatrix} 2310.34 & 286.84 & 181.56 \\ 286.84 & 3193.26 & 607.81 \\ 181.56 & 607.81 & 3727.71 \end{bmatrix}$$

$$\hat{y} = 1201.946 + \frac{1}{2}\begin{bmatrix}0.5813 & 0.3878 & 0.5095\end{bmatrix}\begin{bmatrix}75.529 \\ 25.662 \\ 36.465\end{bmatrix}$$

$$\beta^{-1} = \begin{bmatrix} -0.01425 & -0.001769 & -0.0011196 \\ -0.001769 & -0.01969 & -0.003748 \\ -0.0011196 & -0.003748 & -0.02299 \end{bmatrix}$$

and

$$b = \begin{bmatrix}75.529 \\ 25.662 \\ 36.465\end{bmatrix}$$

Therefore the stationary point was obtained as

$$x_t = -\frac{1}{2}\beta^{-1}b = -\frac{1}{2}\begin{bmatrix} -0.01425 & -0.001769 & -0.0011196 \\ -0.001769 & -0.01969 & -0.003748 \\ -0.0011196 & -0.003748 & -0.02299 \end{bmatrix}\begin{bmatrix}75.529 \\ 25.662 \\ 36.465\end{bmatrix} = \begin{bmatrix}0.5813 \\ 0.3878 \\ 0.5095\end{bmatrix}$$

The stationary points were in coded form and in reference to equation (5), they were transformed into their actual values as

$$x_1 = \frac{X_1 - 100}{20} \Rightarrow 0.5813 = \frac{X_1 - 100}{20} \quad \therefore X_1 = 111.63$$

$$x_2 = \frac{X_2 - 10}{5} \Rightarrow 0.3878 = \frac{X_2 - 10}{5} \quad \therefore X_2 = 11.9$$ and

$$x_3 = \frac{X_3 - 5}{1} \Rightarrow 0.5095 = \frac{X_3 - 5}{1} \quad \therefore X_3 = 5.5$$

Where X_1, X_2 and X_3 are actual variables for nitrogen fertilizer, goat manure and vegimax folia respectively. This implies that at the stationary point the values of nitrogen fertilizer, goat manure and vegimax folia were 111.63 Kg / ha, 11.9 tonnes / ha and 5.5 ml / 20 litres of water respectively.

At the stationary point, the surface response could be predicated through the use of equation (12) such that

$$\hat{y} = \hat{\beta}_0 + \frac{1}{2}x_t^T b$$

$$\hat{y} = 1201.946 + \frac{1}{2}\begin{bmatrix}0.5813 & 0.3878 & 0.5095\end{bmatrix}\begin{bmatrix}75.529 \\ 25.662 \\ 36.465\end{bmatrix}$$

$$\hat{y} = 1201.946 + 36.218$$

$$\hat{y} = 1238.16 \text{ grams}$$

A summary of the optimal settings for nitrogen fertilizer, goat manure and vegimax folia leading to optimal sorghum yield was presented in Table 7.

TABLE 7: OPTIMAL CONDITIONS FOR OPTIMUM SORGHUM GRAIN YIELD.

Variables	Description	Actual value
X_1	Nitrogen fertilizer	112 Kg / ha
X_2	Goat manure	12 tonnes / ha
X_3	Vegimax folia	5.5 ml / 20 litres
$\hat{y}$	Grain yield	4.95 tonnes / ha

On optimization of the response, the findings (Table 7) revealed that 112 Kg/ha of Nitrogen fertilizer supplied as CAN 26%, 12 tons/ha of goat manure and 5.5 ml/20L(of water) of Vegimax folia were the optimum values that led to maximum sorghum grain yield of 4.95 tons/ha.

Nature of the Stationary Point

This study determined the stationary point and then assessed whether it was a saddle point (i.e., a point of inflection), a maximum, a minimum, or both. R-statistical software was used to automatically produce the eigenvalues for this investigation, and the results are shown in table 8.

Table: 8 Eigenvalues for the Stationary Point

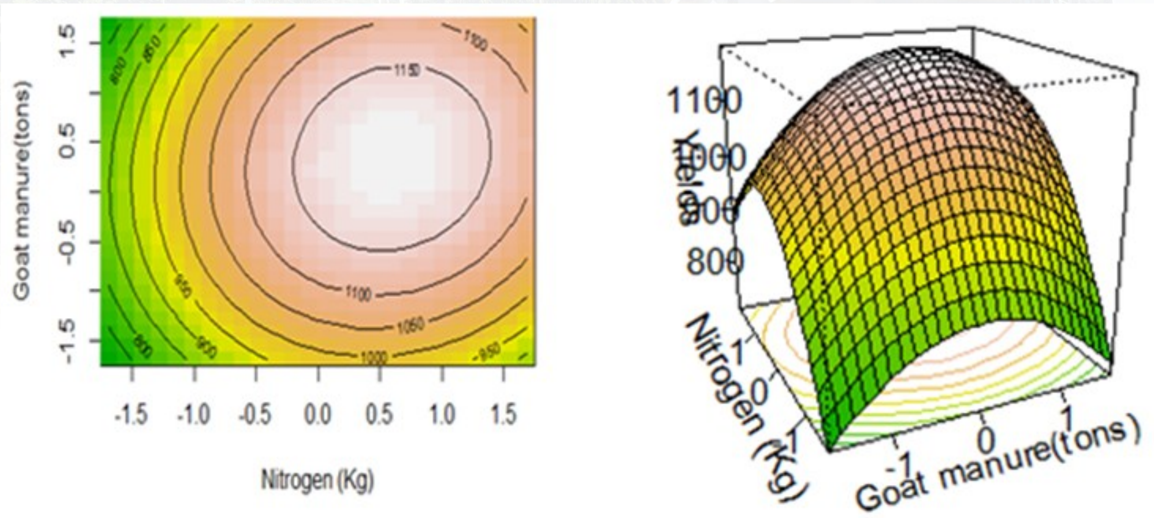
$values		
-38.83478	-57.30332	-72.87467

The features of the stationary point can be determined from the signs of the respective eigenvalues. If all of the eigenvalues have positive signs, the stationary point is a location of least reaction; if all of them have negative signs, a location of maximum response; and if their signs differ, an inflection point. The stationary point must have been where there was the greatest response because the eigenvalues were negative.

Response Surface and Contour Plots

Contour plots give the researcher a graphical representation of the response surface that makes it easier to describe the surface's form and pinpoint the optimum with good accuracy. The response surfaces and associated contour plots were produced by the study using the R-Statistical programme. Plots for various combinations of variables (nitrogen fertilizer, goat manure, and vegimax folia) are shown in Figures 1, 2, and 3, together with the accompanying trend of variance in the response (sorghum grain yield) within the chosen range of input variables.

Figure 1, depicts the relationship between goat dung and nitrogen fertilizer on sorghum grain output. The figure indicates that the production of sorghum grain yield was positively impacted by goat dung and nitrogen fertilizer. This shows that raising the amount of goat dung and nitrogen fertilizer from low to high will boost the yield of sorghum grain up to a certain point. This is so because both goat dung and nitrogen fertilizer are abundant in crucial plant nutrients, particularly nitrogen, which promotes sorghum growth and production.

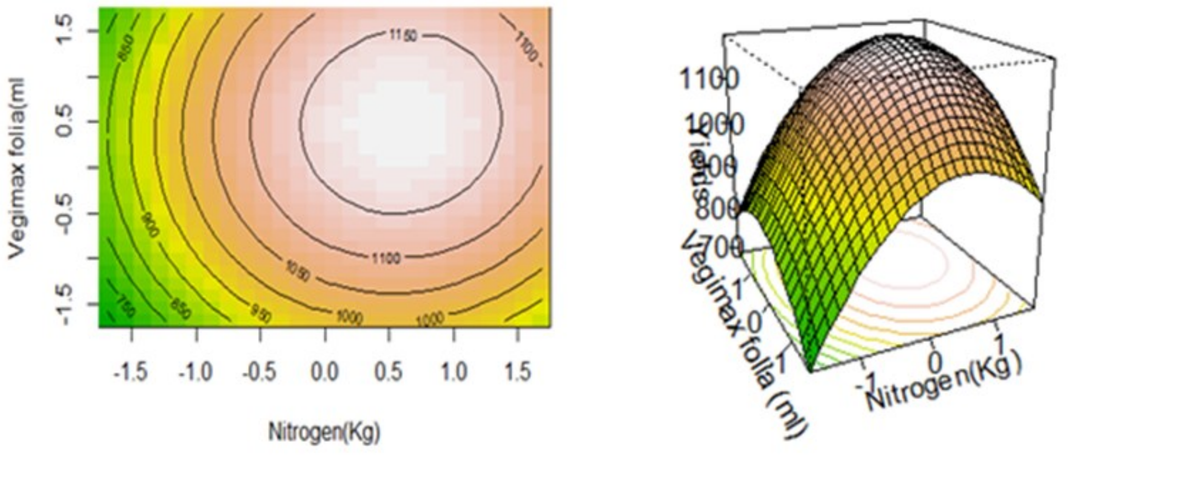


(a) Contour plot with Vegimax folia kept constant

(b) Constant Response surface plot with Vegimax folia kept constant

Figure1. A Contour Plot and a Response Surface Illustrating a Surface with Vegimax Folia Kept Constant

was noted that increasing nitrogen fertilizer and vegimax folia to moderate levels increased the production of sorghum grain yield. Further increase of both nitrogen fertilizer and vegimax folia leads to decrease in the production of sorghum grain. Just like nitrogen fertilizer, vegimax folia are rich in nitrogen, phosphorous and potassium that are absorbed directly by the plant through the surface. Therefore the study attributed the increased sorghum grain production to the essential elements in nitrogen fertilizer and vegimax folia.

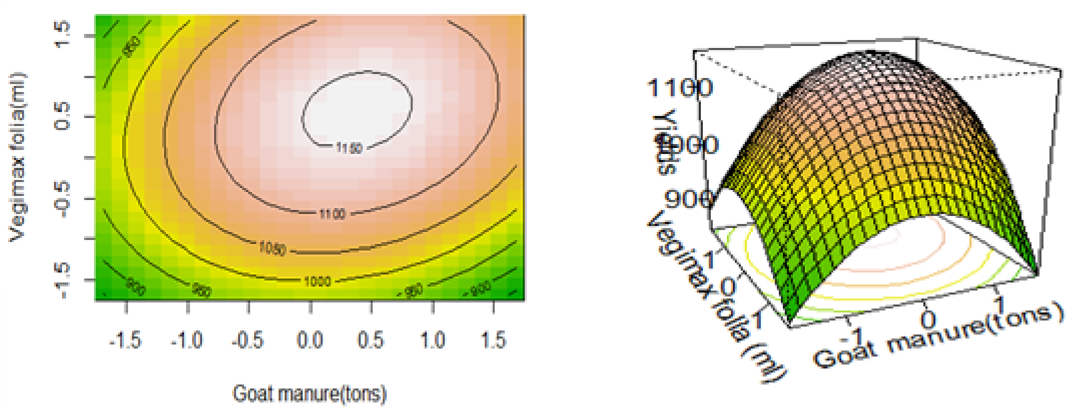


(a) Contour plot with Goat manure kept constant

(b) Response surface with Goat manure kept constant

Figure 2. A Contour Plot and a Response Surface Illustrating a Surface With Goat Manure Kept Constant

Figure 3 shows a surface plot of sorghum grain yield as a function of goat manure and vegimax folia. It was noted that goat manure and vegimax folia have a direct effect on the production of sorghum grain yield up to a certain level and then the sorghum grain yield decreases with further increase of both goat manure and vegimax folia. The three response surface plots reveal that high production of sorghum grain yield is favoured by applying moderately high levels of nitrogen fertilizer, goat manure and vegimax foli



(a) Contour plot with nitrogen fertilizer kept constant

(b) Response surface with nitroger fertilizer kept constant

Figure 3. A Contour Plot and a Response Surface Illustrating a Surface with Nitrogen Fertilizer Kept Constant

CONCLUSION AND RECOMMENDATIONS

There was significant relationship between nitrogen fertilizer, goat manure, vegimax folia and the sorghum yield. The second-order model developed was valid and at a confidence level of 0.95, the model adequately explained at least 86.3 % of the variations on the sorghum response to nitrogen fertilizer, goat manure and vegimax folia. The optimal settings for nitrogen fertilizer, goat manure and vegimax folia that lead to a maximum sorghum yield of 4.95 tons per hectare were 112 Kg / ha, 12 tons / ha and 5.5 ml / 20 litres respectively. The study recommend that the model be applied by farmers in predicting the sorghum response to nitrogen fertilizer, goat manure and vegimax folia. With the optimal settings for nitrogen fertilizer, goat manure and vegimax folia identified for maximum sorghum yield, the study recommend that farmers and agricultural practitioners adopt these settings in their sorghum cultivation practices. The study also recommends that outreach and education programs be designed to disseminate the information to farmers, ensuring they understand and can apply these optimal settings effectively.

CONFLICT OF INTEREST

The authors declare no conflict of interest

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REFERENCES

Aczel, A. D., & Sounderpandian, J. (1999). *Complete business statistics*. Boston, MA: Irwin/ McGraw Hill

Box, G. E., & Wilson, K. B. (1951). On the experimental attainment of optimum conditions. Journal of the Royal Statistical Society: Series B (Methodological), 13(1), 1-38

Chelule, J. C. C. (2014). *Application of Response Surface Methodology in Modelling a Farm Production Process in the Presence of Random -Effects* (Doctoral dissertation)

Kagwiria, D. (2019). Effect of organic and inorganic fertilizer regimes on grain sorghum growth and yield (Doctoral dissertation, University of Nairobi).

Khuri, A. I., & Mukhopadhyay, S. (2010). Response surface methodology. *Wiley Interdisciplinary Reviews: Computational Statistics*, *2* (2), 128-149

Masai, K. L. (2019). *Application of Response Surface Methodology in Modelling and Optimization of The Yields of Common Bean (Phaseolus Vulgaris L.) using Animal Organic Manures* (Doctoral dissertation)

Montgomery, D. C. (2014). Stu Hunter's contributions to experimental design and quality engineering. *Quality Engineering*, *26*(1), 5-15

Muriithi, D. K. (2015). Application of response surface methodology for optimization of potato tuber yield. *American Journal of Theoretical and Applied Statistics*, *4*(4), 300-304

Muui, C. W., Muasya, R. M., & Kirubi, D. T. (2013). Identification and evaluation of sorghum (Sorghum bicolor (L.) moench) germplasm from Eastern Kenya. *African Journal of Agricultural Research*, *8*(36), 4573-4579

Mwadalu, R., Mochoge, B., Mwangi, M., Maitra, S., & Gitari, H. (2022). Response of Gadam sorghum (Sorghum bicolor) to farmyard manure and inorganic fertilizer application. *International Journal of Agriculture, Environment and Biotechnology*, *15*(1), 51-60

Myers, R. H., Montgomery, D. C., & Anderson-Cook, C. M. (2016). *Response surface methodology: process and product optimization using designed experiments*. John Wiley & Sons.

Myers, R. H., Montgomery, D. C., Vining, G. G., Borror, C. M., & Kowalski, S. M. (2004). Response surface methodology: a retrospective and literature survey. *Journal of quality technology*, *36*(1), 53-77

Singari, Ranganath & vp, Vipin & Harshit, (2015). Surface Roughness Prediction Model for CNC Turning of EN-8 Steel Using Response Surface Methodology. IJETAE. 5. 135-143

Titirmare, N. S., Ranshur, N. J., Patil, A. H., Patil, S. R., & Margal, P. B. (2023). Effect of Inorganic Fertilizers and Organic Manures on Physical Properties of Soil: A Review. *International Journal of Plant & Soil Science*, *35*(19), 1015-1023.

Wambugu, S. M, (2011). Increasing Sorghum utilization and marketability through food product diversification. Available on: https://www.kirdi.go.ke/index.php/more/files/category/4-researchreport?download=7:asareca-sorghum-project-kenya-end-of-project-report-20th-september-2011. Accessed on April 10th 2018