EFFECTS OF HALL CURRENT, ROTATION AND INCLINED MAGNETIC FIELD ON A FLUID FLOWING OVER POROUS PARALLEL VERTICAL PLATES WITH HEAT AND MASS TRANSFER

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ABSTRACT
An unsteady incompressible Magnetohydrodynamics (MHD) boundary flow of an electrically conducting fluid between two parallel vertical plates was considered. A strong, steady and inclined magnetic field of strength $\vec{B}$ was applied into the fluid region. The coupled non-linear partial differential equations governing the flow were first non-dimensionalized then solved using the finite difference method. Numerical values were simulated using the Matlab program. The profiles for velocity, temperature and concentration were demonstrated graphically for various values of the parameters m^* for Hall current, Ω for angular velocity of the system, S for the suction velocity at the plate and λ for the Sine of the angle α of application of the magnetic field. The results were then interpreted physically to provide important insights in geophysical fluid dynamics. It was found that the primary velocity decreases at various rates from maximum at the left plate to zero away from the plate for various values of parameters $S$and λ .As distance from the left plate increases, the secondary velocity decreases to a critical value then increases to a maximum constant value for various values of parameters m^*, Ω, λ and S. The temperature and concentration decreases at various rates from a maximum value at the left plate to zero away from the plate for various values of S.
Keywords: Magnetohydrodynamics, Heat transfer, Mass transfer, Coriolis Effect.

1. INTRODUCTION

The study of MHD boundary layer flow has many engineering applications such as MHD generators, flow meters and controlling flow of molten metal in metallurgical industries. The effects of rotation on the hydromagnetic free convection flow of an incompressible viscous and electrically conducting fluid past a uniformly accelerated infinite isothermal vertical plate in the presence of heat and mass transfer was studied by Muthucumaraswamy *et al.*(2010). The magnetic field was taken to be normal to the plate. It was shown that the velocity increased with increasing values of thermal Grashof and mass Grashof number.

Ahmed and Sinha (2014) extended the work of Muthucumaraswamy *et al.* (2010) by considering a three-dimensional MHD time dependent flow of a viscous incompressible fluid induced by uniformly accelerated motion of an infinite vertical plate with uniform mass diffusion in a rotating fluid under the influence of a uniform magnetic field applied perpendicularly to the flow. The primary fluid motion is decelerated under the action of transverse magnetic field while the temperature falls due to low thermal conductivity and low diffusivity.

Hossain *et al.* (2015)) studied the effects of Hall and ion-slip currents on the unsteady MHD flow through a viscous incompressible electrically conducting fluid with the oscillations of an infinite non-conducing vertical porous plate in a rotating system. The Sherwood distribution increased with increase of Hall and ion-slip parameters while it decreased with increase in heat source and magnetic parameters. Purkayastha and Choudhury (2014) analyzed the effects of Hall current and thermal radiation with first-order chemical reaction on elastico-viscous fluid on a rotating porous channel with suction and injection. The velocity profile showed an enhancement trend in the neighborhood of the plate and then followed a decreasing path. Garg *et al*.(2014) however considered rotation and Hall current effects on MHD convective flow of a second grade fluid through a porous medium in a porous vertical channel in slip-flow regime with thermal radiation.

Thamizhsudar *et al*. (2015) considered Hall current effects on a MHD flow of an exponentially accelerated plate relative to a rotating fluid with uniform temperature and mass diffusion. The results of the study showed that temperature of the plate decreased with increase in values of the Prandtl number while the concentration near the plate increased with decrease in values of Schmidt number. Kinyanjui *et al*. (1998) studied MHD natural convection flow of a viscous incompressible rotating fluid with Hall current and viscous dissipative heat. The study found that increasing the rotation parameter lowered the primary velocity and raised the secondary velocity profiles.

Research combining Hall current, rotating system and inclined magnetic field was carried out by Sarkar *et al.* (2014). The study investigated Hall effects on MHD flow in a rotating channel in the presence of an inclined magnetic field and found that Hall currents and angle of inclination retarded the primary velocity. The work however considered the flow through a rotating channel and did not incorporate the effects of heat and mass transfer. Suresh and Manglik (2014) analyzed the Hall current and inclined magnetic field effects on Hartman flow of an electrically conducting fluid in a porous medium over a rotating plate. Laplace transform technique was used to solve the dimensionless partial differential equations governing the flow. The primary velocity was found to slightly increase as Hall parameter was increased. The secondary velocity was observed to increase with increase in the Hall parameter. When rotation parameter was reduced to zero, primary velocity became maximum but decreased as rotation parameter was increased.

From literature review, combined effects of Hall current, rotation and inclined magnetic field on a free convection fluid flow with heat and mass transfer has not been considered on a single model. The drawing of strips of polymers through a die is carried out through a stagnant cooling fluid. The quality of the strips is found to depend on the rate of heat and mass transfer on the stretching surface. Compromise on quality of the final product is what makes the study of the combined effects of Hall current, rotation and uniform inclined magnetic field on a free convection fluid flow with heat and mass transfer to be necessary. This study therefore endeavors to investigate the effects of Hall current, rotation and inclined uniform magnetic field on free convection flow of a fluid past vertical porous plates with heat and mass transfer.

2. MODEL FORMULATION

An unsteady incompressible MHD boundary layer flow of an electrically conducting fluid between two porous parallel vertical plates is considered. The fluid is permeated by a constant magnetic field of strength $\vec{B}$ applied at an angle $\propto$ to the plates. The plates are located on the planes $x^+ = -L$and $x^+ = L$. A second fluid is injected uniformly from the left plate and there is uniform suction from the right plate with velocity $u_o{}^+$ applied at $t^+ > 0$ as shown on Fig.1.The system rotates with uniform angular velocity $\overrightarrow{\Omega^+}$ about the x^+ axis.

At $t^+ > 0$, the left plate starts moving upwards with velocity U_o in its own plane. The physical variables are functions of x^+ and t^+ only as both plates are infinite in y^+ and z^+.

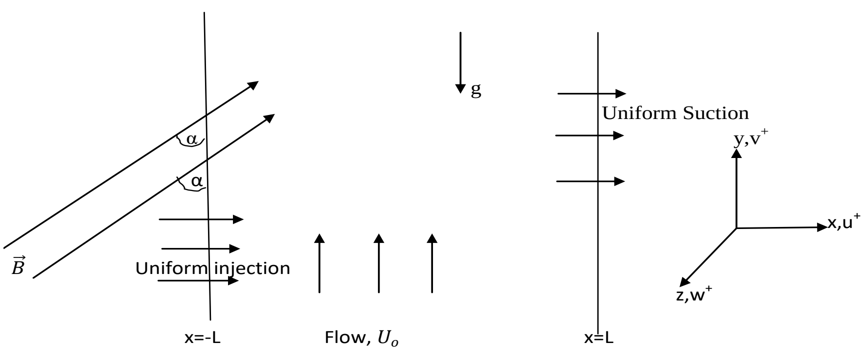

Figure 1: Flow Configuration

Under these conditions, the unsteady flow is governed by the following equations:

$$\frac{\partial v^+}{\partial t^+} + u_o{}^+ \frac{\partial v^+}{\partial x^+} - 2\Omega^+ w^+ = \upsilon \frac{\partial^2 v^+}{\partial x^{+2}} + g\beta(T^+ - T_1{}^+) + g\beta^* (C^+ - C_1{}^+) + \frac{\sigma \mu_e{}^2 \lambda^2 H_o{}^2 (m^* w^+ \lambda - v^+)}{\rho(1 + m^{*2} \lambda^2)} \tag{2.1}$$

$$\frac{\partial w^+}{\partial t^+} + u_0{}^+\frac{\partial w^+}{\partial x^+} + 2\Omega^+ v^+ = \upsilon\frac{\partial^2 w^+}{\partial x^{+2}} - \frac{\sigma\mu_e{}^2\lambda^2 H_o{}^2(m^* v^+\lambda + w^+)}{\rho(1+m^{*2}\lambda^2)} \tag{2.2}$$

$$\frac{\partial T^+}{\partial t^+} + u_0{}^+\frac{\partial T^+}{\partial x^+} = \frac{k}{\rho c_p}\frac{\partial^2 T^+}{\partial x^{+2}} + \frac{\upsilon}{c_p}\left[\left(\frac{\partial v^+}{\partial x^+}\right)^2 + \left(\frac{\partial w^+}{\partial x^+}\right)^2\right] + \frac{\sigma\mu_e{}^2\lambda^2 H_o{}^2(v^{+2}+w^{+2})}{\rho c_p(1+m^{*2}\lambda^2)} \tag{2.3}$$

$$\frac{\partial C^+}{\partial t^+} + u_0{}^+\frac{\partial C^+}{\partial x^+} = D\frac{\partial^2 C^+}{\partial x^{+2}} \tag{2.4}$$

The initial conditions are:

$$\left.\begin{aligned} v^+(-L^+,0) = w^+(-L^+,0) = 0 \\ v^+(L^+,0) = w^+(L^+,0) = 0 \\ T^+(-L^+,0) = T^+(L^+,0) = T_1{}^+ \\ C^+(-L^+,0) = C^+(L^+,0) = C_1{}^+ \end{aligned}\right\} \text{at } t^+ \le 0 \tag{2.5}$$

The boundary conditions are:

$$\left.\begin{aligned} v^+(-L,t^+) = U_0 \\ w^+(-L^+,t^+) = 0 \\ v^+(L^+,t^+) = 0 \\ w^+(L^+,t^+) = 0 \\ T^+(-L^+,t^+) = T_2{}^+ \\ T^+(L^+,t^+) = T_1{}^+ \\ C^+(-L^t,t^+) = C_2{}^+ \\ C^+(L^+,t^+) = C_1{}^+ \end{aligned}\right\} \text{at } t > 0. \tag{2.6}$$

To non-dimensionalize equations (2.1) to (2.4), the initial conditions (2.5) and the boundary conditions (2.6), the following scaling quantities are applied.

$$v = \frac{v^+}{U_0},\ w = \frac{w^+}{U_0},\qquad t = t^+\frac{U_o{}^2}{\upsilon},\qquad \Omega = \frac{\Omega^+\upsilon}{U_o{}^2},\qquad M^2 = \frac{\sigma\mu_e{}^2 H_o{}^2\upsilon}{\rho U_o{}^2}$$

$$y = y^+\frac{U_o}{\upsilon},\qquad Gr = \frac{\upsilon g\beta\,(T_2{}^+ - T_1{}^+)}{U_o{}^3},\qquad Ec = \frac{U_o{}^2}{C_p(T_2{}^+ - T_1{}^+)},\ Pr = \frac{\upsilon\rho C_p}{k} = \frac{\mu C_p}{k}$$

$$Gc = \frac{\upsilon g\beta\,(C_2{}^+ - C_1{}^+)}{U_o{}^3},\qquad C = \frac{C^+ - C_1{}^+}{C_2{}^+ - C_1{}^+},\quad \theta = \frac{T^+ - T_1{}^+}{T_2{}^+ - T_1{}^+},\qquad S = \frac{u_o{}^+}{U_o},\ Sc = \frac{\upsilon}{D}$$

Equations (2.1) to (2.4) reduce to the following non-dimensional form.

$$\frac{\partial v}{\partial t} + S\frac{\partial v}{\partial x} + 2\Omega w = \frac{\partial^2 v}{\partial x^2} + Gr\theta + GcC + \frac{M^2\lambda^2(m^* w - \lambda v)}{1+m^{*2}\lambda^2} \tag{2.7}$$

$$\frac{\partial w}{\partial t} + S\frac{\partial w}{\partial x} - 2\Omega v = \frac{\partial^2 w}{\partial x^2} - \frac{M^2\lambda^2(m^* v + \lambda w)}{1+m^{*2}\lambda^2} \tag{2.8}$$

$$\frac{\partial \theta}{\partial t} + S\frac{\partial \theta}{\partial x} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial x^2} + Ec\left[\left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right] + \frac{M^2\lambda^2 Ec(v^2+w^2)}{1+m^{*2}\lambda^2} \tag{2.9}$$

$$\frac{\partial C}{\partial t} + S\frac{\partial C}{\partial x} = \frac{1}{Sc}\frac{\partial^2 C}{\partial x^2} \tag{2.10}$$

The initial conditions in non-dimensional form are:

$$\left.\begin{array}{c} v(-L,0) = w(-L,0) = 0 \\ v(L,0) = w(L,0) = 0 \\ \theta(-L,0) = \theta(L,0) = 0 \\ C(-L,0) = C(L,0) = 0 \end{array}\right\} \text{at } t \leq 0. \tag{2.11}$$

The boundary conditions in non-dimensional form are:

$$\left.\begin{array}{c} v(-L,t) = 1 \\ w(-L,t) = 0 \\ v(L,t) = 0 \\ w(L,t) = 0 \\ \theta(-L,t) = 1 \\ \theta(L,t) = 0 \\ C(-L,t) = 1 \\ C(L,t) = 0 \end{array}\right\} \text{at } t > 0 \tag{2.12}$$

3. SOLUTION OF THE MODEL EQUATIONS

To solve the dimensionless governing equations, the finite difference method is applied to give:

$$\begin{aligned} v(i,j+1) &= \frac{\Delta t}{(\Delta x)^2}[v(i-1,j) - 2v(i,j) + \\ &v(i+1,j)] + \Delta t[Gr\theta(i,j) + GcC(i,j)] - \\ &S\frac{\Delta t}{\Delta x}[v(i+1,j) - v(i,j)] - \left[\frac{\Delta t M^2 \lambda^2}{1+m^{*2}\lambda^2} - 1\right] v(i,j) + \\ &\Delta t\left[\frac{m^* M^2 \lambda^3}{1+m^{*2}\lambda^2} - 2\Omega\right] w(i,j) \end{aligned} \tag{3.1}$$

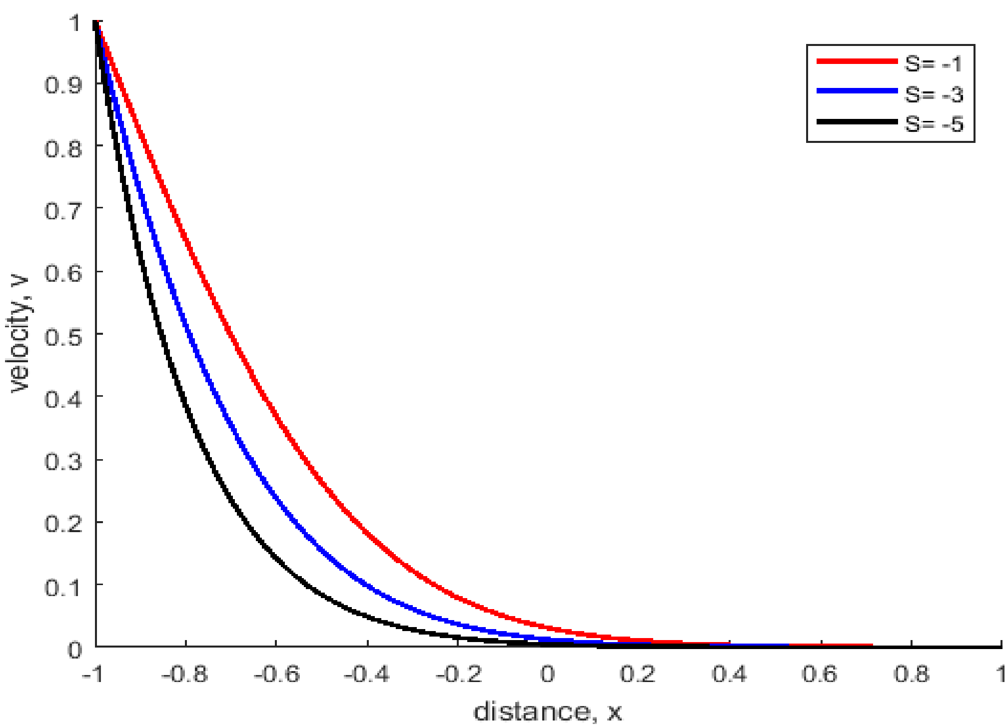

Figure 2. Primary velocity profiles for different values of S

$$w(i,j+1) = \frac{\Delta t}{(\Delta x)^2}[w(i-1,j) - 2w(i,j) + w(i+1,j)] - S\frac{\Delta t}{\Delta x}[w(i+1,j) - w(i,j)] - \left[\frac{\Delta t M^2 \alpha^2}{1+m^{*2}\alpha^2} - 2\Omega\Delta t - 1\right]w(i,j) - \left[\frac{\Delta t m^* M^2 \lambda^3}{1+m^{*2}\lambda^2}\right]v(i,j) \tag{3.2}$$

$$\theta(i,j+1) = \frac{\Delta t}{Pr(\Delta x)^2}[\theta(i-1,j) - 2\theta(i,j) + \theta(i+1,j)] - S\frac{\Delta t}{\Delta x}\theta(i+1,j) + \left[S\frac{\Delta t}{\Delta x} + 1\right]\theta(i,j) + Ec\frac{\Delta t}{(\Delta x)^2}[\{v(i+1,j) - v(i,j)\}^2 + \{w(i+1,j) - w(i,j)\}^2] + Ec\frac{\Delta t M^2 \lambda^2}{1+m^{*2}\lambda^2}[v^2(i,j) + w^2(i,j)] \tag{3.3}$$

$$C(i,j+1) = \frac{\Delta t}{Sc(\Delta x)^2}[C(i-1,j) - 2C(i,j) + C(i+1,j)] - S\frac{\Delta t}{\Delta x}C(i+1,j) + \left[S\frac{\Delta t}{\Delta x} + 1\right]C(i,j) \tag{3.4}$$

Subject to the initial conditions

$$\left.\begin{array}{l} q(-40,0) = q(40,0) = 0 \\ \theta(-40,0) = \theta(40,0) = 0 \\ C(-40,0) = C(40,0) = 0 \end{array}\right\} \quad j \leq 0 \tag{3.5}$$

And boundary conditions

$$\left.\begin{array}{l} q(-40,j) = 1 \\ q(40,j) = 0 \\ \theta(-40,j) = 1 \\ \theta(40,j) = 0 \\ C(-40,j) = 1 \\ C(40,j) = 0 \end{array}\right\} \quad j > 0 \tag{3.6}$$

4. RESULTS AND DISCUSSION

To better understand the effects of Hall current, rotation, suction velocity and angle of inclination of the applied magnetic field on the primary and secondary velocities, temperature and concentration, the following parameter values were used: $Sc = 0.6$, $Ec = 0.02$, $Gr = -0.5$, $Pr = 0.71$, $Gc = 10$ and $M^2 = 5$. The results were represented in graphs as in fig. 2 to fig. 9. From fig. 2, 8 and 9, increase in the Suction Parameter lowers the primary velocity, the temperature and concentration of the fluid. It however raises the secondary velocity as shown in figure 7. From fig. 3 and fig. 6, increase in the angle of application of the magnetic field lowers both the primary and the secondary velocity profiles. This agrees with Seth *et.al.* (2009).

From fig. 4, increase in the Hall current parameter increases the secondary velocity. The rotation parameter reduces the secondary velocity as shown in fig. 5. This implies that rotation has a retarding effect on the fluid flow in the tranverse direction as was observed by Situma *et al.* (2015).

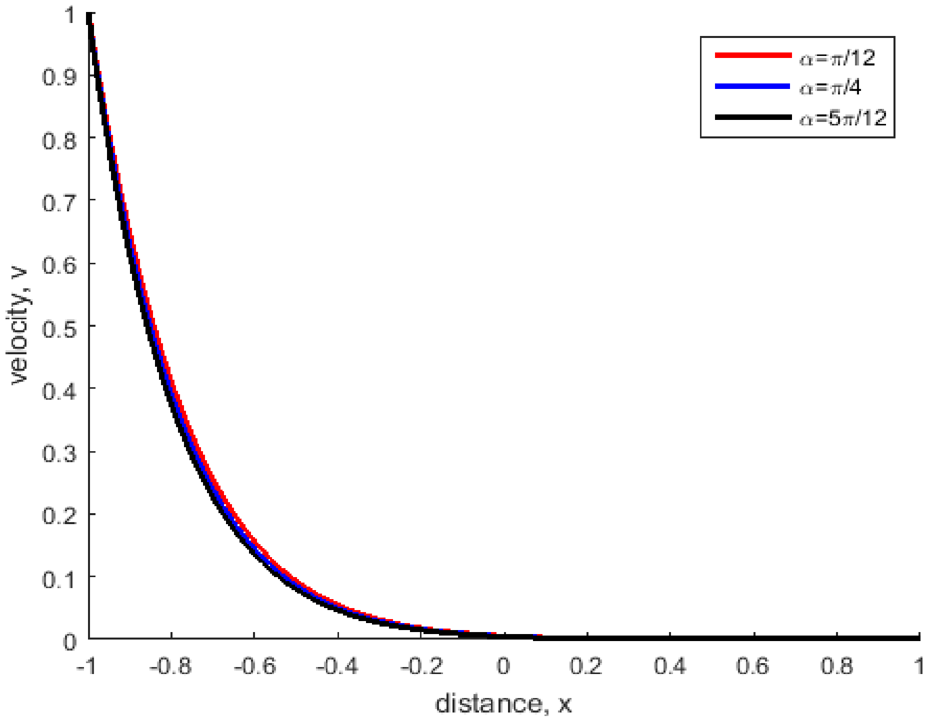

Fig. 3. Primary velocity profiles for different values of $\alpha = \sin^{-1}\lambda$

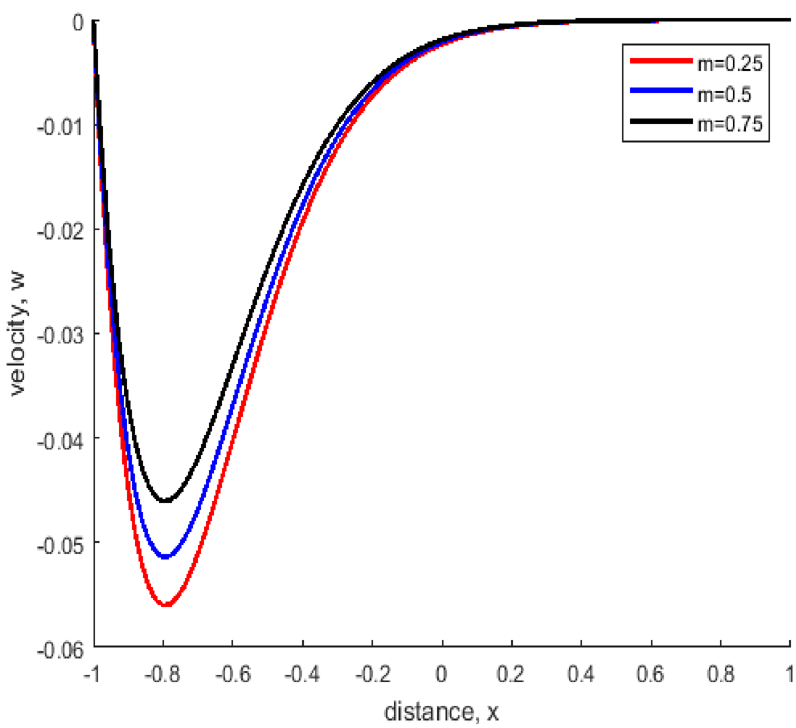

Figure 4: Secondary velocity profiles for different values of m^*.

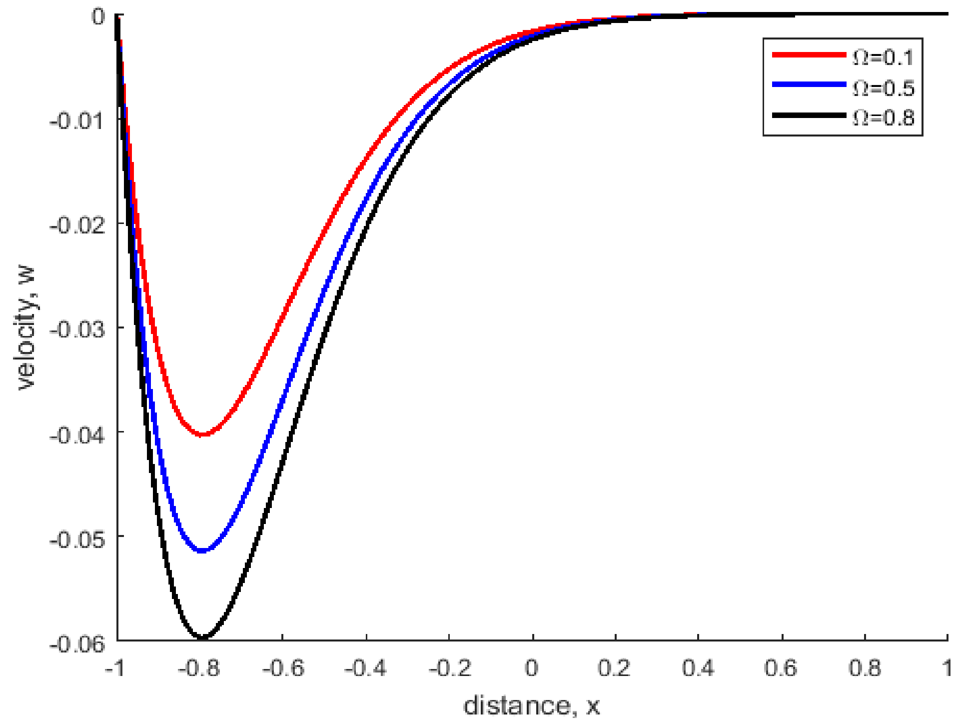

Figure 5 Secondary velocity profiles for different values of Ω.

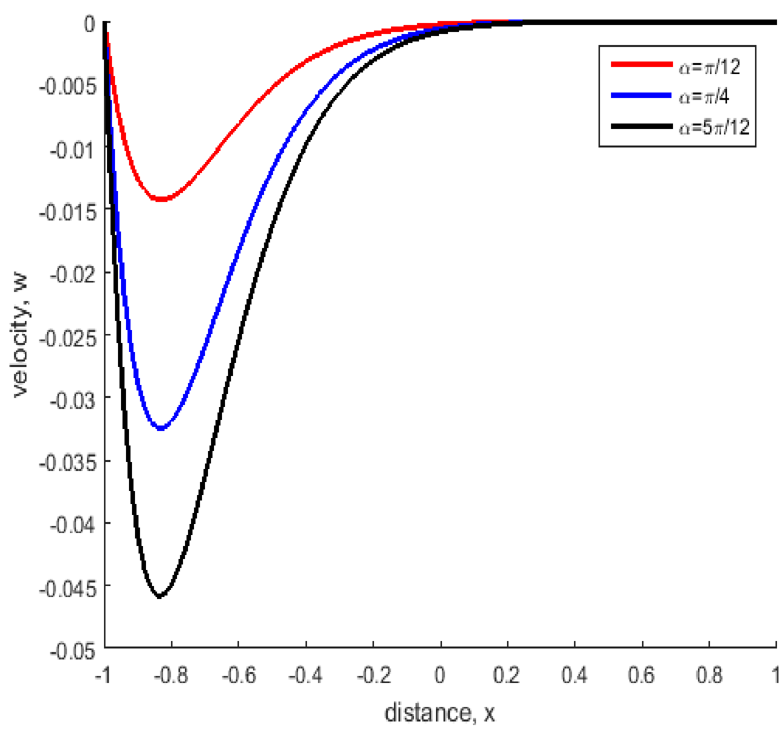

Fig. 6. Secondary velocity profiles for different values $\alpha = \sin^{-1}\lambda$

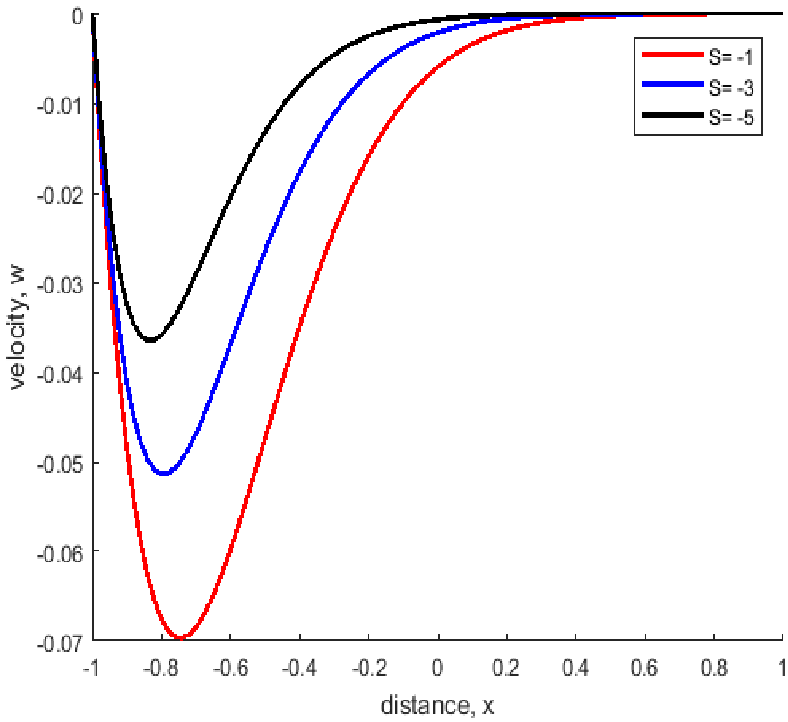

Figure 7. Secondary velocity profiles for different values of S.

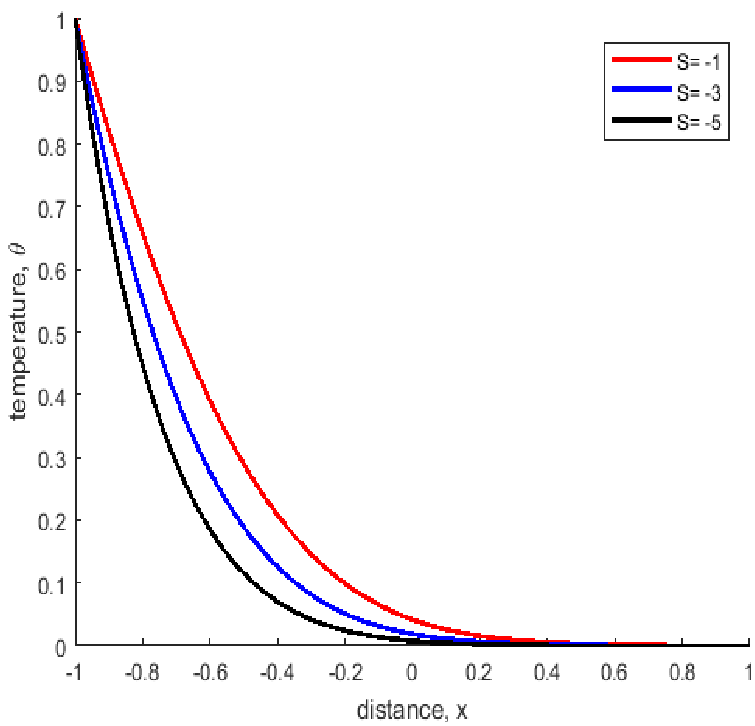

Figure 8. Temperature profiles for different values of S.

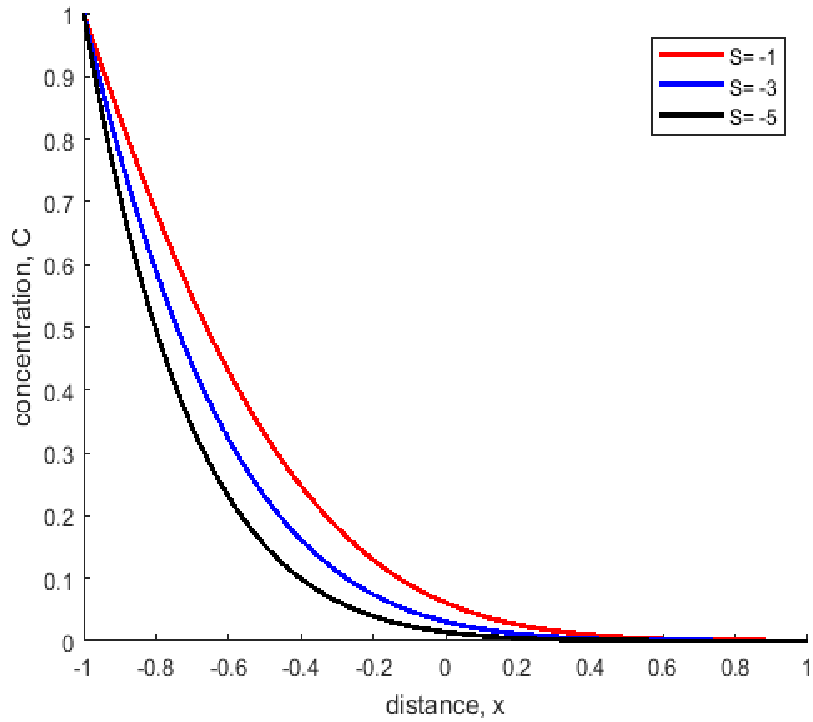

Figure 9. Concentration profiles for different values of S.

CONCLUSION

This study involved the effects of Hall current, rotation and inclined magnetic field on a fluid flowing over porous parallel vertical plates with heat and mass transfer.

The primary velocity decreases with increasing values of suction parameter or angle of application of the magnetic field. The secondary velocity increases when either the Hall parameter increases or the suction parameter increases. The secondary velocity increases when either the rotation parameter or the angle of application of the magnetic field decreases.

The temperature of the plate increases with decreasing values of the suction parameter.

The concentration of the fluid increases with decreasing values of the suction parameter. The primary velocity, temperature and concentration take maximum values at the plate surface but reduce to zero far away from the plate. The secondary velocity is maximum at the plate but reduces to a minimum before rising to a steady maximum far away from the plate.

Nomenclature

$(\vec{u}, \vec{v}, \vec{w})$	Non-dimensional velocity components
$(\vec{u}^+, \vec{v}^+, \vec{w}^+)$	Dimensional velocity components
t	Non- dimensional time
t^+	Dimensional time
Θ	Non-dimensional temperature
θ^+	Dimensional temperature
T_1^+	Wall temperature
T_2^+	Fluid temperature at infinity
(x, y, z)	Cartesian Co-ordinates
T	Non-dimensional temperature
$\vec{U}_\infty$	Free stream velocity
$\vec{g}$	Acceleration due to gravity
C_p	Specific heat Capacity at constant pressure
σ	Electrical conductivity of the fluid
υ	Kinematic Coefficient of viscosity
ρ	Fluid density
$\vec{q}$	Fluid velocity
λ	Sine of the angle of the applied magnetic field
α	Angle of the applied magnetic field
Gr	Grashof number
Gc	Mass Grashof number
Pr	Prandtl number
Ec	Eckert number
μ_e	Magnetic permeability of the medium
$\vec{B}$	Applied Magnetic field
M	Magnetic field parameter

m^*	Hall Current parameter
Ω	Angular velocity of the fluid
Sc	Schmidt number
D	Diffusion Coefficient
S	Suction parameter
β	Volumetric coefficient of thermal expansion
C	Non-dimensional fluid concentration
C_1^+	Concentration at the plate
C_2^+	Concentration far from the plate

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